Commitment in Repeated Games with Incomplete Information

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Model

Consider a two-player repeated game with lack of information on one side.

- The finite set of states is K with prior $p_0 \in \Delta(K)$.
- Players take actions in A, B. Payoffs are $u^k(a,b)$, $v^k(a,b)$.
- Monitoring is perfect.
- Cheap-talk messages are allowed.
- Denote $U_\delta(\sigma,\tau)$, $V_\delta(\sigma,\tau)$ the expected discounted payoffs under the strategies σ,τ. Similarly $U_n(\sigma,\tau)$, $V_n(\sigma,\tau)$ the average payoffs in the n-stage game, and $U_\infty(\sigma,\tau)$, $V_\infty(\sigma,\tau)$ the undiscounted limit payoffs (using a Banach limit).

Commitment

We want to study the best payoff that player 1 can achieve by committing ex-ante to a strategy σ. We call commitment payoff the following quantities.

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- $C_n(p_0) = \sup\{U_n(\sigma, \tau) : \sigma, \tau \in BR_n(\sigma)\}$.
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- One can define the uniform commitment payoff.
Questions:
- Can we characterize the commitment payoff?
- For the informed/uninformed player?

Revelation principle

Since player 1 has full commitment power, he can act as a mediator à la Myerson-Forges.

- More precisely, consider the auxiliary game where the payoff of player 1 is set to 0. Consider the set of communication equilibrium outcomes (distributions over states and plays) of this auxiliary game.
- Then the commitment payoff is the maximal expected average (actual) payoff of player 1 over this set.
- The consequence is that w.l.o.g., player 1 recommends action to player 2 (using the cheap talk messages). Because of perfect monitoring, deviations are detected with probability 1.

Benchmark 1: Complete information

Assume here $|K| = 1$.
Let C^* be the maximal payoff of player 1 over the set of feasible payoffs which are individually rational for player 2.

$$C^* = \max\{u(x) : x \in \Delta(A \times B), v(x) \geq \min \max v\}$$

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$C_\infty = \lim C_n = \lim C_\delta = C^*$

- Obviously player 1 cannot get more.
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- Extends to n-player games?

Benchmark 2: Commitment by the uninformed player

Assume now incomplete information and that player 2 is the informed player. Define

$$C_*(p) = \max \sum_k p^k u^k(x^k)$$

s.t.

$$\forall k, l, v^k(x^k) \geq v^k(x^l)$$

and

$$\forall q \in \Delta(K), \sum_k q^k v^k(x^k) \geq V(q)$$

with $V(q) = \min_\alpha \max_\beta \sum_k q^k v^k(\alpha, \beta)$.

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Player 1 acts as a mediator and asks player 2 to report the state. Then he offers an incentive compatible contract. Deviations are punished by approachability. The conditions are necessary for all versions of the repeated game.

Commitment by the informed player: A Cav u formula

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- $C_\infty(p_0) = \text{Cav}\, f(p_0)$. Fixing σ and a best-reply τ, the martingale of a posteriori converges. The asymptotic distribution of actions x is non-revealing. If $\sum_k p^k v^k(x) < \text{Vex}\, V(p)$, player 2 would deviate.

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- for $p < \frac{3}{4}$, $f(p) = 0$;
- for $p > \frac{3}{4}$, $f(p) = 1$.

Cav $f(\frac{1}{2}) = \frac{2}{3} f(\frac{3}{4}^{+}) + \frac{1}{3} f(0) = \frac{2}{3}$.

Example 2: selling information (related to Hörner-Skrzypacz)

Consider the following game with two states 0 and 1 (prior $\frac{1}{2}$) and three actions for player 2, b_0, b_1, D. The payoffs are:

$$\begin{array}{cccc} & b_0 & b_1 & D \end{array}$$

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$$(1-p)v^0(x)+pv^1(x) = (1-p)x(b_0)+px(b_1)-mx(D) \leq \max\{p, 1-p\}-mx(D)$$

So $(1-p)v^0(x) + pv^1(x) \geq \max\{p, 1-p\}$ implies $x(D) = 0$.

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Thus $f(p) = 0$.

Example 2: ctd'

Consider now the discounted game with high δ and fix T. Consider the following strategy.

- Say nothing before stage T.
- Then reveal fully the state at stage $T+1$ only if player 2 played D at the first T stages. Otherwise say nothing ever after.

Player 2 is promised a payoff of 1 after stage T. The punishment level is $\frac{1}{2}$ when player 1 is silent. Player 2 is willing to obey if:

$$-(1-\delta^T)m+\delta^T \geq \frac{1}{2}$$

that is $(1-\delta^T) \leq \frac{1/2}{1+m}$. Player 1 can thus get approximately $\frac{m}{2(1+m)}$.

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This is optimal: $C_\delta \leq \frac{m}{2(1+m)}$ for each δ.

An optimization problem

A strategy pair (σ, τ) induces a process $(x_t, p_t)_t$ with values in $\Delta(B) \times \Delta(K)$ such that (p_t) is a martingale starting at p_0.

x_t is the distribution of recommended actions used by player 1 at stage t and p_t is the belief of player 2 at stage t after receiving the recommended action.

Denote $\mathcal{F}_T$ the information filtration of player 2 (the recommended actions).

The incentive constraint for player 2 at stage T is:

$$IC_T : \mathbb{E}[\sum (1-\delta)\delta^{t-1} v^k(x_{t+T}) \mid \mathcal{F}_T] \geq V(p_T) \; a.s.$$

We have then

$$C_\delta(p_0) = \sup\{\mathbb{E}[\sum (1-\delta)\delta^{t-1} u^k(x_t)] \;\; s.t. \;\; IC_T(\forall T)\}$$

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Claim

$$\lim C_\delta(p_0) = \lim C_n(p_0) = \sup \mathbb{E}[\int_0^1 u^k(x_t)dt]$$

s.t. $\forall T$, $\mathbb{E}[\int_T^1 v_k(x_t)dt \mid \mathcal{F}_T] \geq (1-T)V(p_T)$, *with* (x_t, p_t) *process such that* (p_t) *is a martingale starting at* p_0.

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In the zero-sum case $v = -u$, the value of the program:

$$\sup \mathbb{E}[\textstyle\int_0^1 u^k(x_t)dt] \text{ over martingales s.t. } \forall T,$$
$$\mathbb{E}[\textstyle\int_T^1 -u_k(x_t)dt \mid \mathcal{F}_T] \geq (1-T)\text{Vex}\,(-U)(p_T),$$

is just Cav $U(p_0)$.

An optimization problem

On the example,

$$C_\delta(p_0) \leq C_1 := \sup\{\mathbb{E}[\sum(1-\delta)\delta^{t-1}u^k(x_t)] \;\; s.t. \;\; IC_1\}$$

$C_1 = \sup\{\mathbb{E}[\sum(1-\delta)\delta^{t-1}u^k(x_t)] \text{ s.t. } \mathbb{E}[\sum(1-\delta)\delta^{t-1}v^k(x_t) \mid b_1] \geq \frac{1}{2}\}.$

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Denote F the expected discounted frequency of the dominated action D. We have $\mathbb{E}[\sum(1-\delta)\delta^{t-1}u^k(x_t)] = mF$ and

$$\mathbb{E}[\sum(1-\delta)\delta^{t-1}v^k(x_t)] \leq -mF + (1-F)$$

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So $C_1 \leq \max\{mF : -mF + (1-F) \geq \frac{1}{2}\} = \frac{m}{2(1+m)}$.

Full revelation with delay

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On the selling information example, this gives the following optimization problem:

$$G(p) = \max\{mt : -mt + (1-t) \geq \max\{p; 1-p\}\}$$

This gives $G(p) = \frac{m}{1+m}(1 - \max\{p; 1-p\})$ which is concave in p.

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Remark: This is increasing in m. If several paiements are possible, it is best to enforce the largest one.

Full revelation with delay

On the investment game

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this gives,

$$\max\{t + \frac{1-t}{2} : -t + \frac{1-t}{2} \geq 0\} = \frac{2}{3} = \text{Cav}\, f(\frac{1}{2})$$

Full revelation with delay

Denote for each k, $v_k^* = \max_b v^k(b)$ and $u_k^* = \max\{u^k(b) : v^k(b) \geq v_k^*\}$.

By using full revelation with delay, player 1 guarantees the quantity $G(p)$:

$$\max\{t \sum_k p^k u^k(x) + (1-t) \sum_k p^k u_k^* : t \sum_k p^k v^k(x) + (1-t) \sum_k p^k v_k^* \geq V(p)\}$$

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$G(p)$ need not be concave!

Two revelation steps

Consider the following game (prior $\frac{1}{2}$).

$$\begin{array}{cccc} b_0 & b_1 & D_0 & D_1 \end{array}$$

$$k = 0 : [(0,1); (0,0); (1,-1); (-1,-1)]$$

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Player 1 can only get 0 with a single step of revelation.

- If player 1 reveals at time 0, player 2 never plays D_0, D_1.
- If player 1 is silent at time 0 and reveals the state at time T, the expected payoff is 0 before time T (and after also).

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$G(p)$ is symetric around $1/2$ and maximal at $1/4$ and $3/4$:
$G(1/4) = G(3/4) = 1/16$. So Cav $G(1/2) = 1/16$.

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Player 1 first splits, then waits, then reveals.

Pending questions

- Is full revelation with delay enough when player 1's payoff does not depend on the state?
- Do we have $C(p) = \text{Cav}\, G(p)$ for some class of games?
- Can we bound the number of revelation steps?