Bayesian Games with Multiple Priors

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Introduction

Classical Sender-Receiver game:

- Incomplete information on one side, only the Sender is informed of the state of the nature.
- The Sender sends a message to the Receiver, who takes an action.
- Both utilities of the Sender and the Receiver depend on the state of the nature and of the Receiver's action.
- Both the Sender and the Receiver share the same prior belief over the state of nature.

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However, in some environnements, agents have to take into account several prior beliefs. We study a general model with multiple priors which encompasses two particular cases:

(1) Sender-Receiver game with ambiguity: the Receiver has a set of prior beliefs.

(2) Communication game between a Sender and several Receivers who have different priors.

The general setup

We consider a reduced-form model, where the sender's payoff depends directly on profiles of posterior beliefs.

- Consider a single agent, the Sender.
- A finite set of states of the nature Ω.
- Prior belief $p^* \in \Delta(\Omega)$ of the agent.
- A finite set of signals (messages) $S = \{1, \ldots, |S|\}$ that the agent can choose.
- Agent's strategy: $\pi : \Omega \to \Delta(S)$.

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- Agent's strategy: $\pi : \Omega \to \Delta(S)$.
- Let I be an index set, e.g. $I = \{1, \ldots, n\}$ or $I = [0, 1]$.
- Let $(p_i)_{i \in I}$ be a profile of prior beliefs, with $p_i \in int\Delta(\Omega)$ for each $i \in I$. We write p_i^s for the posterior of p_i conditional on the signal s, i.e.,

$$p_i^s(\omega) = \frac{\pi(s|\omega)p_i(\omega)}{\sum_{\omega \in \Omega} \pi(s|\omega)p_i(\omega)},$$

if $\sum_{\omega \in \Omega} \pi(s|\omega)p_i(\omega) > 0$, and arbitrary otherwise.

The general setup (II)

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- We assume that the agent commits to his signaling strategy π before learning the state of the nature: equivalently, the Receiver observes π.
- The Sender's program is then:

$$\max_{\pi:\Omega\to\Delta S} \sum_{s,\omega} u((p_i^s)_{i\in I}, \omega)\pi(s|\omega)p^*(\omega), \qquad (\mathcal{P})$$

subject to

$$p_i^s(\omega) = \frac{\pi(s|\omega)p_i(\omega)}{\sum_{\omega\in\Omega}\pi(s|\omega)p_i(\omega)},$$

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Application 1: Persuading an audience

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$$A^*((p_i^s)_{i\in I}) = \arg\max_{a\in A}\min_{i\in I}\sum_{\omega\in\Omega} v(a,\omega)p_i^s(\omega).$$

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- Particular case: different priors (same prior $\Rightarrow$ Kamenica and Gentzkow, 2011).

Application 2: Bayesian persuasion with ambiguity (II)

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  - In practice, there are not as it is too costly to reproduce the experimental tests.

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- Authorities need to approve or not whether the drug can be put on the market, in the presence of uncertainties.
  - Precautionary principle ⇒ maxmin preferences (Gilboa and Schmeidler, 89).

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  - In practice, there are not as it is too costly to reproduce the experimental tests.
- Authorities need to approve or not whether the drug can be put on the market, in the presence of uncertainties.
  - Precautionary principle $\Rightarrow$ maxmin preferences (Gilboa and Schmeidler, 89).
- Drugs companies are biased.
  - They want their drugs to be approved "regardless what."
  - Except perhaps if the effectiveness of the drug is very low (reputational effect).

An example

There are three states of the nature, ω_1, ω_2 and ω_3. The Sender has two possible messages s and s', and the Receiver has two possible actions a and b. The payoffs are the following:

(v, u)	a	b
ω_1	1,1	0,0
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Interpretation:

- Action a = the national agency gives the authorization of selling the drug.
- Action b = the national agency forbids the selling of the drug.
- State ω_1 = the drug is effective and new.
- State ω_2 = the drug is not effective.
- State ω_3 = the drug is effective, but is similar to many others drugs already sold (hence the national agency is indifferent between giving the authorization or not).

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Thus, no communication is required for the Sender to achieve his highest payoff.

An example (III)

(v,u)	a	b
ω_1	1,1	0,0
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Second case: ambiguity on one side. Assume now that the Receiver has a set of priors given by

$$P = \left\{ (p_1, p_2, p_3) \ : \ p_1 = \frac{1}{3},\ p_2 \in \left[\frac{1}{6}, \frac{1}{2}\right],\ p_3 = \frac{2}{3} - p_2 \right\}.$$

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Ex-ante, the optimal action of the Receiver is b:

$$\max_{x\in[0,1]} \min_{p_2\in[\frac{1}{6},\frac{1}{2}]} \left[\frac{1}{3}x + p_2(1-x) + \frac{2}{3} - p_2\right] = \max_{x\in[0,1]} \left[-\frac{1}{6}x + \frac{2}{3}\right]$$

$$\Rightarrow x = 0$$

An example (IV)

(v, u)	a	b
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Suppose now that the Sender sends the message s when the state is ω_3, and the message s' when the state is either ω_1 or ω_2.

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Suppose now that the Sender sends the message s when the state is ω_3, and the message s' when the state is either ω_1 or ω_2.

The sets of posteriors are $\{(0,0,1)\}$ conditional on s, and

$$\left\{(q_1,q_2,q_3)\ :\ q_1=\frac{\frac{1}{3}}{\frac{1}{3}+p_2},\ q_2=1-q_1,\ q_3=0,\ p_2\in\left[\frac{1}{6},\frac{1}{2}\right]\right\},$$

conditional on s'.

An example (V)

(v,u)	a	b
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- Conditional on receiving the message s', the Receiver solves:

$$\max_{x\in[0,1]} \min_{p_2\in[\frac{1}{6},\frac{1}{2}]} \left[\frac{\frac{1}{3}}{\frac{1}{3}+p_2}x + \frac{p_2}{\frac{1}{3}+p_2}(1-x)\right] = \max_{x\in[0,1]} \begin{cases} \frac{1}{3}+\frac{1}{3}x & \text{if } x<\frac{1}{2} \\ \frac{1}{2} & \text{if } x=\frac{1}{2} \\ \frac{3}{5}-\frac{1}{5}x & \text{if } x>\frac{1}{2} \end{cases}$$

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$$\Rightarrow x = 1.$$

Hence, with this strategy, the Sender guarantees himself his highest payoff 1.

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Conclusion:

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$\Rightarrow$ More information is revealed with ambiguity in this example.

(ii) The Sender gets a higher payoff when the Receiver has the set of priors P, than when he has a unique prior $p = (\frac{1}{3}, \frac{1}{2}, \frac{1}{6}) \in P$.

However, there exists priors in P, e.g. $(\frac{1}{3}, \frac{1}{6}, \frac{1}{2})$, such that the Sender gets the same payoff with or without ambiguity.

Splitting procedure

For two $|\Omega|$-dimensional vectors p and q, define the $|\Omega|$-dimensional vector q/p as follows: $(q/p)(\omega) = q(\omega)/p(\omega)$ if $p(\omega) > 0$, $(q/p)(\omega) = 0$ if $p(\omega) = 0$.

Proposition 1

Let S be a finite set and $(p_i^s)_{i \in I}$ be a set of posterior beliefs for each $s \in S$. The following statements are equivalent.

1. There exists a signaling function $\pi : \Omega \to \Delta(S)$ such that p_i^s is the posterior of p_i, given s, i.e.,

$$p_i^s(\omega) = \frac{\pi(s|\omega)p_i(\omega)}{\sum_\omega \pi(s|\omega)p_i(\omega)},$$

   whenever $\sum_\omega \pi(s|\omega)p_i(\omega) > 0$, for all (i, s).

2. There exists $(\lambda_i^s)_{i,s}$, with $\lambda_i^s \in [0, 1]$ and $\sum_s \lambda_i^s = 1$ for all (i, s), such that

   (i) $(1, \dots, 1) \in \sum_s \lambda_i^s (p_i^s/p_i)$ for all (i, s).

   (ii) $\lambda_i^s (p_i^s/p_i) = \lambda_j^s (p_j^s/p_j)$ for all (i, j, s).

Splitting procedure: remarks

- Classic splitting result: if I is a singleton, then (2)(ii) is vacuously satisfied, while (2)(i) simply means that p_i is in the convex hull of the p_i^s (see Aummann and Maschler, 1967).

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- More generally, a necessary condition for splitting multiple priors is that each prior p_i is included in the convex hull of the set of posteriors $\{p_i^s : s \in S\}$, for each $i \in I$. However, this is not sufficient.

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- With two states of the nature: condition (2)(ii) of Proposition 1 implies that for all (p_i, p_j) and ω, if $p_i(\omega) \geq p_j(\omega)$, then $p_i^s(\omega) \geq p_j^s(\omega)$ (and conversely if $p_i(\omega) \leq p_j(\omega)$).

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- Conditions (2)(i) and (2)(ii) are geometric conditions in the space of likelihood ratios.
    - Condition (2)(i): the unit vector is in the convex hull of the likelihood ratios $(p_i^s/p_i)_s$ for each i
    - Condition (2)(ii): the likelihood ratios $(p_i^s/p_i)_i$ are collinear for each s

Splitting procedure: example

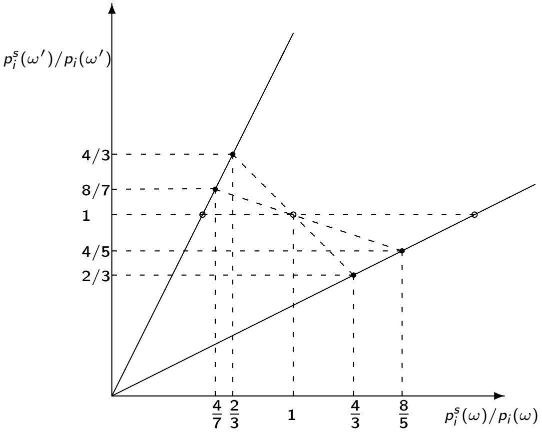
Let $\Omega = \{\omega, \omega'\}$ be the set of states of the nature, p_1 and p_2 two prior beliefs, and s_2 and s_2 two signals.

For concreteness, let $p_1 = (1/2, 1/2)$ and $p_2 = (1/4, 3/4)$.

There exists a signaling function that induces the posteriors $(p_1^{s_1}, p_2^{s_1}) = ((1/3, 2/3), (1/7, 6/7))$ and $(p_1^{s_2}, p_2^{s_2}) = ((2/3, 1/3), (2/5, 3/5))$.

Indeed, the likelihood ratios are given by $(p_1^{s_1}/p_1) = (2/3, 4/3)$, $(p_1^{s_2}/p_1) = (4/3, 2/3)$, $(p_2^{s_1}/p_2) = (4/7, 8/7)$, $(p_2^{s_2}/p_2) = (8/5, 4/5)$, and it is easy to verify that condition (2)(i) and (2)(ii) are satisfied with $(\lambda_1^{s_1}, \lambda_1^{s_2}) = (1/2, 1/2)$ and $(\lambda_2^{s_1}, \lambda_2^{s_2}) = (7/12, 5/12)$.

Splitting procedure: example (II)



Splitting procedure: corollary

Corollary

If we choose a splitting of an arbitrary prior, then conditions (2)(i) and (2)(ii) pin down all other posteriors.

- Choose an arbitrary $i^* \in I$, and let $(\lambda_{i^*}^s, p_{i^*}^s)_s$ be a splitting of p_{i^*}.

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- Hence, $(p^s_i)_{i \in I}$ can be viewed as the image of a map $f : \Delta\Omega \to \prod_{i \in I} \Delta\Omega$, where the i-th component is given by

$$f_i(p^s_{i^*}) := \frac{p_i \cdot (p^s_{i^*} / p_{i^*})}{\| p_i \cdot (p^s_{i^*} / p_{i^*}) \|},$$

for some fixed but arbitrary p_{i^*}, $i^* \in I$.

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- In words, $f_i(p^s_{i^*}) \in \Delta\Omega$ is the posterior belief of p_i if the signal s is observed and p_{i^*} is split into $(p^s_{i^*})_s$.

Characterization of the sender's optimal payoff

To ease notation, we denote p the fixed but arbitrary prior p_{i^*}.

Let $U_{p,p^*} : \Delta\Omega \to \mathbb{R}$ be the function, parameterized by (p, p^*), defined by

$$U_{p,p^*}(q) := \sum_{\omega} u(f(q), \omega) \frac{p^*(\omega)}{p(\omega)} q(\omega),$$

for all $q \in \Delta\Omega$.

Theorem

The sender's optimal payoff is $\mathrm{cav}\, U_{p,p^*}(p)$.

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- The i-th component f_i of the function f is given by

$$f_i(q) := \frac{c_i q}{1 + q(c_i - 1)},$$

with $c_i := \frac{1-p_{i^*}}{p_{i^*}} \frac{p_i}{(1-p_i)}$ for all $i \in I$.

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- The maximization problem then amounts to choose two posterior beliefs $(p_{i^*}^s, p_{i^*}^{s'})$, with $p_{i^*}^s \leq p_{i^*} \leq p_{i^*}^{s'}$, such that

$$\frac{p_{i^*}^{s'} - p_{i^*}}{p_{i^*}^{s'} - p_{i^*}^s} U_{p_{i^*}, p^*}(p_{i^*}^s) + \frac{p_{i^*} - p_{i^*}^s}{p_{i^*}^{s'} - p_{i^*}^s} U_{p_{i^*}, p^*}(p_{i^*}^{s'})$$

is maximized.

Example: partial revelation of information

There are two states of the nature ω_1 and ω_2, and the profile of priors of the Receiver is $\left(\frac{1}{3}+i\right)_{i\in[0,1/3]}$, or equivalently $[\frac{1}{3},\frac{2}{3}]$. The Sender has two possible messages s_1 and s_2, and his prior is $(\frac{1}{2},\frac{1}{2})$. The payoffs are:

state ω_1:

LL	*L*	*M*	*R*	*RR*
0, 10	1, 8	0, 5	1, 0	0, −8

state ω_2:

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If the Sender either fully reveals the state, or does not disclose any information, then his payoff is 0; but he can improve his payoff by partially revealing some information.

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This example is adapted from Aumann and Hart (2003).

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- We can now compute the function $q \mapsto U_{1/3,1/2}(q)$. Whenever the set of posteriors is $[q, f_{1/3}(q)]$, the receiver chooses an action $a^*([q, f_{1/3}(q)])$ that maximizes his worst expected payoff (if there are several, choose one that favors the receiver). For instance, if the set of posteriors is $[1/4, 4/7]$, the receiver choose action M and the sender's payoff is 0.

Example: partial revelation of information (III)

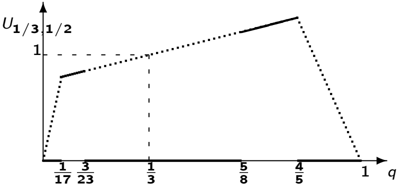

Figure: The payoff function (bold) and its concavification (dots)

If $q = 2/3$, the set of posteriors is $[2/3, 4/9]$ and the optimal action of the receiver is R, which gives a payoff of 1 to the receiver, regardless of the state. However, the sender and the receiver have different prior beliefs and we need to correct for the difference, i.e., we need to compute the term $\sum_\omega \frac{p^*(\omega)}{p(\omega)} q(\omega)$, which is equal to $\frac{3}{4}(1+q)$. Thus, $U_{1/3,1/2}(2/3) = 5/4$.

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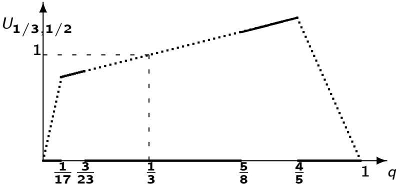

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It follows that the optimal payoff to the sender is 1.

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- Nor can we read from it the sender's payoff following the splitting. Yet, we can learn about the optimal splittings.

- For instance, an optimal splitting is $(1/13, 3/4)$, with induced sets of posteriors $\left[\frac{1}{13}, \frac{1}{4}\right]$ and $\left[\frac{3}{4}, \frac{12}{13}\right]$. If the set of posteriors is $\left[\frac{1}{13}, \frac{1}{4}\right]$ (resp., $\left[\frac{3}{4}, \frac{12}{13}\right]$), the receiver's optimal action is R (resp., L), which guarantees a payoff of 1 to the sender. Partial revelation of information is optimal.

Further research

What happens if both parties have a partial private information on the safety of the drug?

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What if the Sender has some actions?

What if the Receiver can communicate? Do long conversations improve payoffs?

Thank you !